Graduate Certificate in Engineering

Advanced Engineering Mathematics

Bessel Function

* Concept: Bessel functions are solutions to Bessel's differential equation, which arises in various physical problems, such as the vibration of a circular membrane or the propagation of electromagnetic waves in a circular waveguide.
* Related Terms: Bessel's differential equation, modified Bessel functions, spherical Bessel functions, Bessel function of the first kind (Jn(x)), Bessel function of the second kind (Yn(x)), Bessel function of integer order, Bessel function of fractional order.
* Explanation: Bessel functions are canonical solutions to Bessel's differential equation (BDE), which is a second-order ordinary differential equation (ODE) with singular solutions at 0 and infinity. BDE is given by:

$$x^2(d^2y/dx^2) + x(dy/dx) + (x^2 - n^2)y = 0$$

where n is a constant. Bessel functions are usually denoted as Jn(x) (Bessel function of the first kind) or Yn(x) (Bessel function of the second kind). The functions have a series representation, which can be written in terms of the trigonometric functions for integer orders. The Bessel functions of the first kind are finite at the origin, while the Bessel functions of the second kind have a singularity at the origin.

Challenge: Implement a Python function to calculate the value of Jn(x) and Yn(x) for any given x and n.

Eigenvalue and Eigenvector

* Concept: Eigenvalues and eigenvectors are important concepts in linear algebra, which are closely related to matrix transformations.
* Related Terms: Linear transformation, diagonalization, spectral theorem, similarity transformation, determinant, characteristic equation.
* Explanation: Given a square matrix A and a non-zero vector x, if Ax = λx for some scalar λ, then x is called the eigenvector associated with the eigenvalue λ. The eigenvalues are the roots of the characteristic equation, given by det(A - λI) = 0, where I is the identity matrix. The eigenvectors are the non-zero solutions to the homogeneous linear system (A - λI)x = 0. Eigenvalues and eigenvectors are important for understanding the behavior of linear transformations, such as scaling, rotation, or shearing.

Challenge: Prove that the eigenvalues of a triangular matrix are the diagonal elements.

Fourier Series

* Concept: A Fourier series is a representation of a periodic function as a sum of trigonometric functions

with different frequencies and amplitudes.
* Related Terms: Fourier transform, Fourier sine series, Fourier cosine series, coefficients, orthogonal functions.
* Explanation: A Fourier series represents a periodic function f(x) with period 2π as the sum of sines and cosines:

f(x) = a0 + Σ[an * cos(nx) + bn * sin(nx)]

where an and bn are the Fourier coefficients, given by:

an = (1/π) ∫[f(x) * cos(nx) dx], from -π to π

bn = (1/π) ∫[f(x) * sin(nx) dx], from -π to π

The Fourier series converges uniformly to f(x) if f(x) is piecewise continuous and has a piecewise continuous derivative.

Challenge: Use the Fourier series to represent the sawtooth wave function.

Heaviside Function

* Concept: The Heaviside function is a step function, which is defined as 0 for negative arguments and 1 for non-negative arguments.
* Related Terms: Unit step function, impulse function, Dirac delta function, Laplace transform, convolution.
* Explanation: The Heaviside function H(x) is defined as:

H(x) = 0, if x Concept: The Laplace transform is a mathematical tool, which transforms a time-domain function into the frequency domain.
* Related Terms: Inverse Laplace transform, Laplace transform pair, region of convergence, transfer function, convolution theorem, initial value theorem, final value theorem.
* Explanation: The Laplace transform of a function f(t) is defined as:

F(s) = L{f(t)} = ∫[f(t) * e^(-st) dt], from 0 to infinity

where s is a complex variable. The Laplace transform has various properties, such as linearity, differentiation, and integration. The inverse Laplace transform is used to find the time-domain function from its Laplace transform.

Challenge: Use the Laplace transform to solve the second-order differential equation of a mass-spring-damper system.

Matrix

* Concept: A matrix is a rectangular array of numbers, which can be used to represent linear transformations.
* Related Terms: Vector, determinant, inverse matrix, transpose matrix, eigenvalues, eigenvectors, singular value decomposition, rank, nullity, orthogonal matrix, unitary matrix, Hermitian matrix, normal matrix, positive definite matrix.
* Explanation: A matrix A of size m x n is an array of m rows and n columns, where each element is denoted as aij. Matrices can be used to represent linear transformations, such as scaling, rotation, or shearing, by multiplying a vector with a matrix. The determinant of a matrix is a scalar value, which measures the scaling factor of the transformation. The inverse matrix is the multiplicative inverse of a matrix, which is used to find the inverse transformation.

Challenge: Prove that the inverse of a orthogonal matrix is its transpose.

Ordinary Differential Equation (ODE)

* Concept: An ordinary differential equation (ODE) is a mathematical equation, which relates a function and its derivatives.
* Related Terms: Linear ODE, nonlinear ODE, first-order ODE, second-order ODE, higher-order ODE, initial value problem, boundary value problem, existence and uniqueness theorem, Picard's theorem, variation of parameters, Green's function.
* Explanation: An ODE is a mathematical equation that relates a function and its derivatives, such as dy/dx = f(x, y), where f(x, y) is a given function. The order of an ODE is the highest order derivative in the equation. The solution of an ODE is a function that satisfies the equation. The initial value problem is to find the solution of an ODE that satisfies a given initial condition.

Challenge: Solve the first-order linear ODE of a simple RC circuit.

Partial Differential Equation (PDE)

* Concept: A partial differential equation (PDE) is a mathematical equation, which relates a function and its partial derivatives.
* Related Terms: Linear PDE, nonlinear PDE, first-order PDE, second-order PDE, higher-order PDE, initial value problem, boundary value problem, separation of variables, Fourier series, Fourier transform, Green's function, method of characteristics.
* Explanation: A PDE is a mathematical equation that relates a function and its partial derivatives, such as $\partial^2u/\partial x^2 + \partial^2u/\partial y^2 = f(x, y)$, where u(x, y) is a given function. The order of a PDE is the highest order derivative in the equation. The solution of a PDE is a function that satisfies the equation